Learning to Control PDEs with Differentiable Physics

Seminar – Deep Learning in Physics

TUM, Summer Semester 2022

Barnabás Börcsök

Advisor: Lukas Prantl

Overview

- Differentiable Physics
  - Motivation
  - Forward Physics Simulation
  - Differentiable *(“backwards”)* Physics
  - Examples
- Partial Differential Equations (PDEs)
- Results of the paper
- Other examples and outlook
  - Enhancing simulations with Neural Networks (NNs)
  - Inductive Physical Bias

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The big picture – Computer Graphics

Virtual world

Real world

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- Modelling
- Simulating
- Rendering

… the real world

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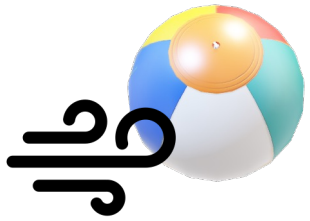

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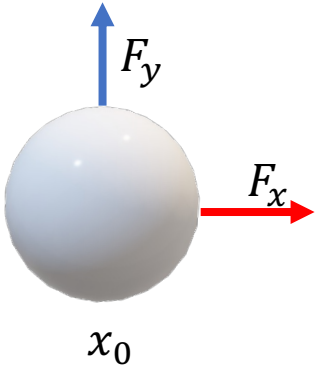

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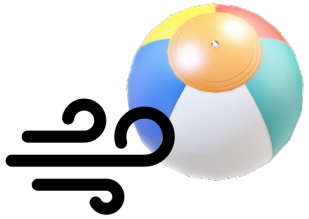

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$$(x_{i+1}, v_{i+1}) = f(x_i, v_i)$$

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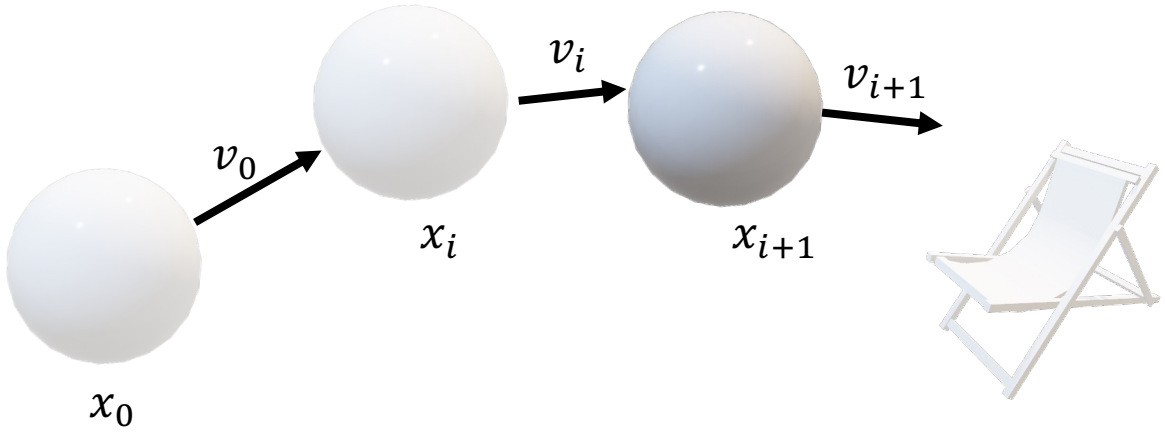

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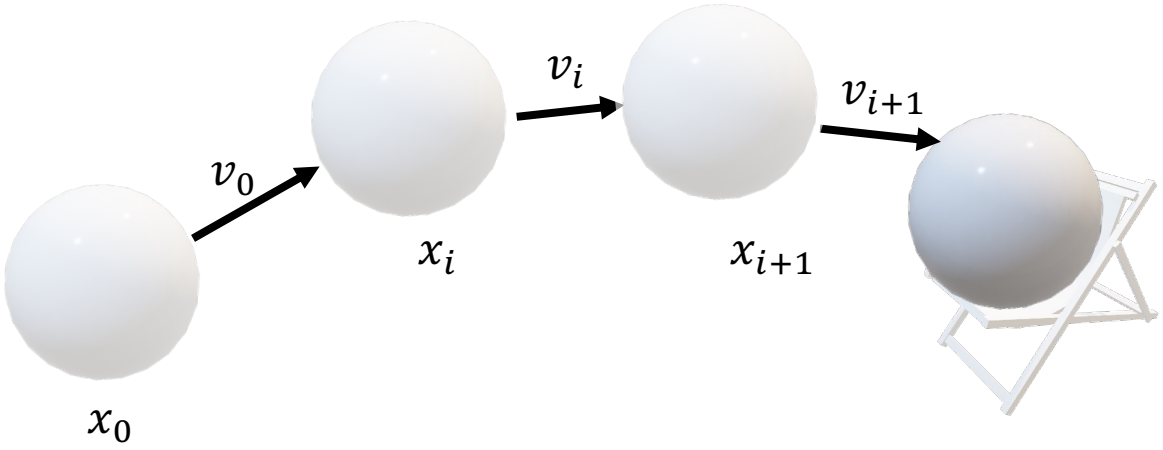

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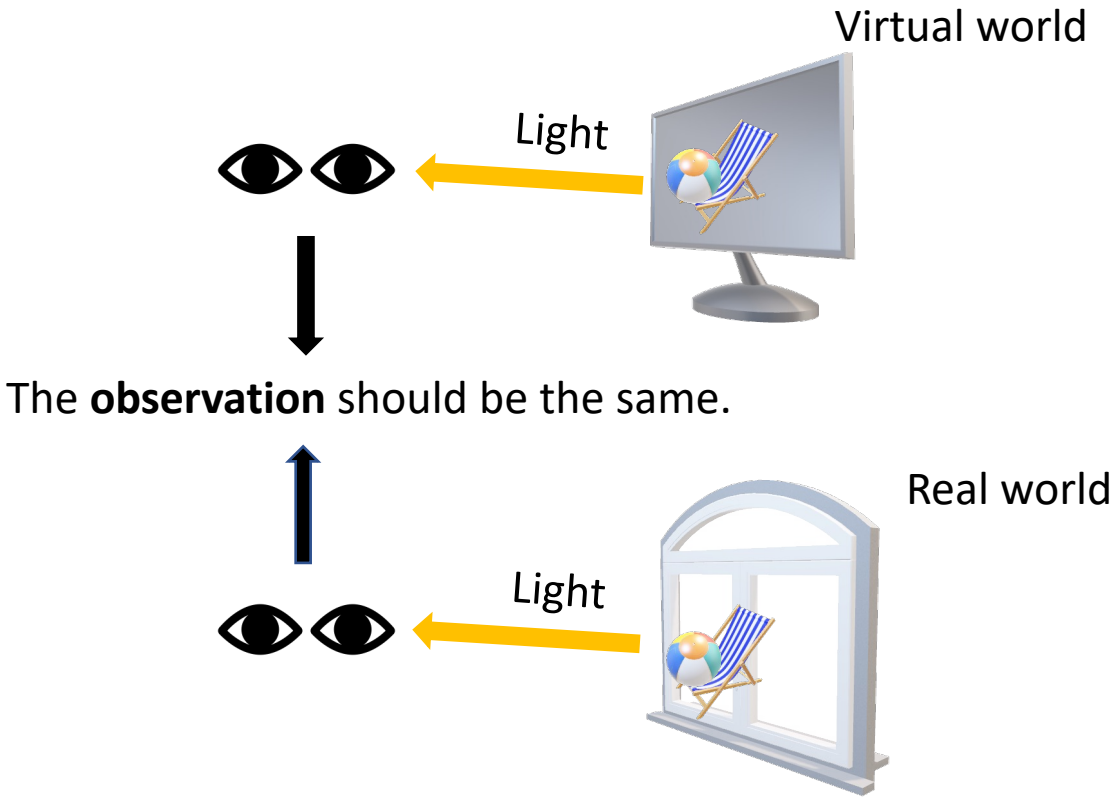
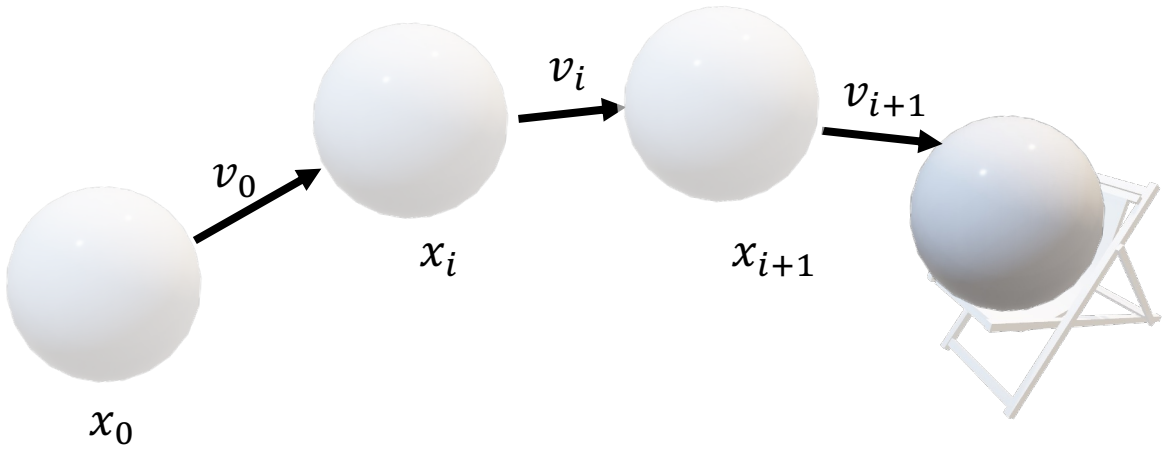


(Forward) Physics Simulation

Physical simulations are much more complex …

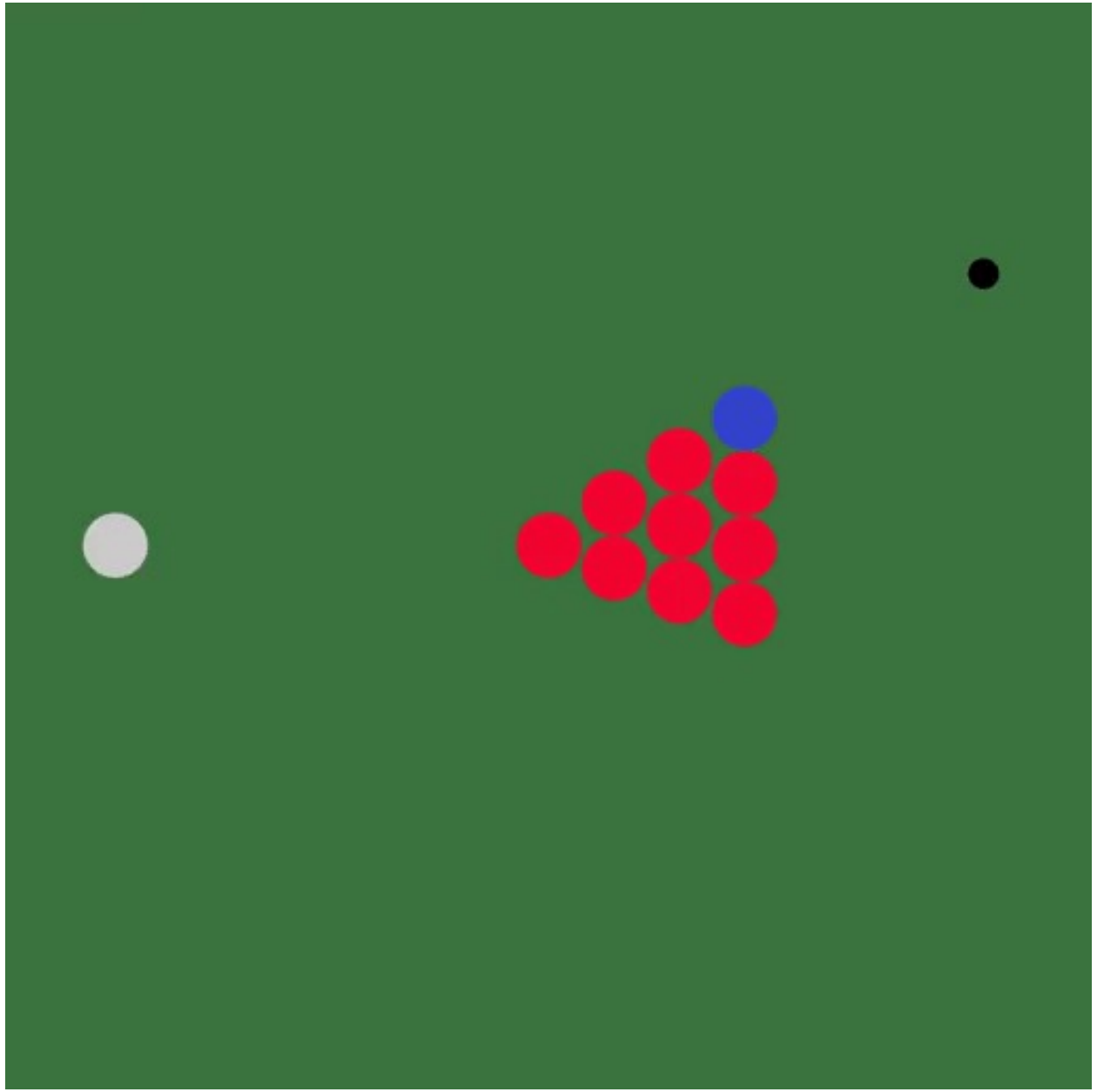
$$(x_{i+1}, v_{i+1}) = f(x_i, v_i)$$



Our Model

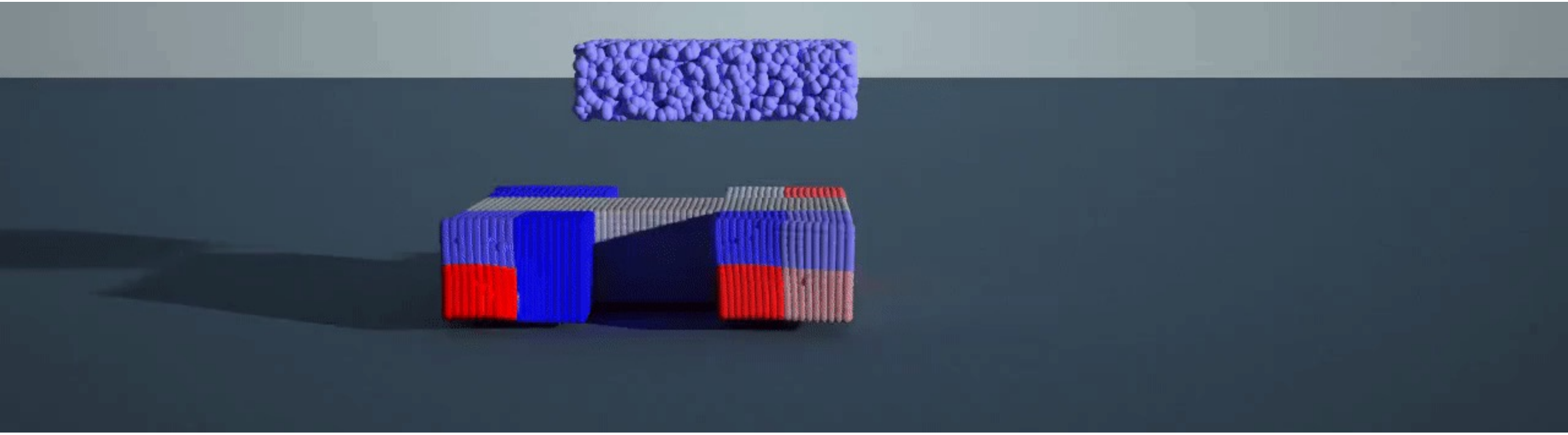
Artistic Simulation of Curly Hair, Hayley et. al

Each strand is made up of springs (Hooke’s law)
Simulating 579 hair strands (44,552 points)



Source: https://github.com/taichi-dev/difftaichi

Elastic Object + Fluid



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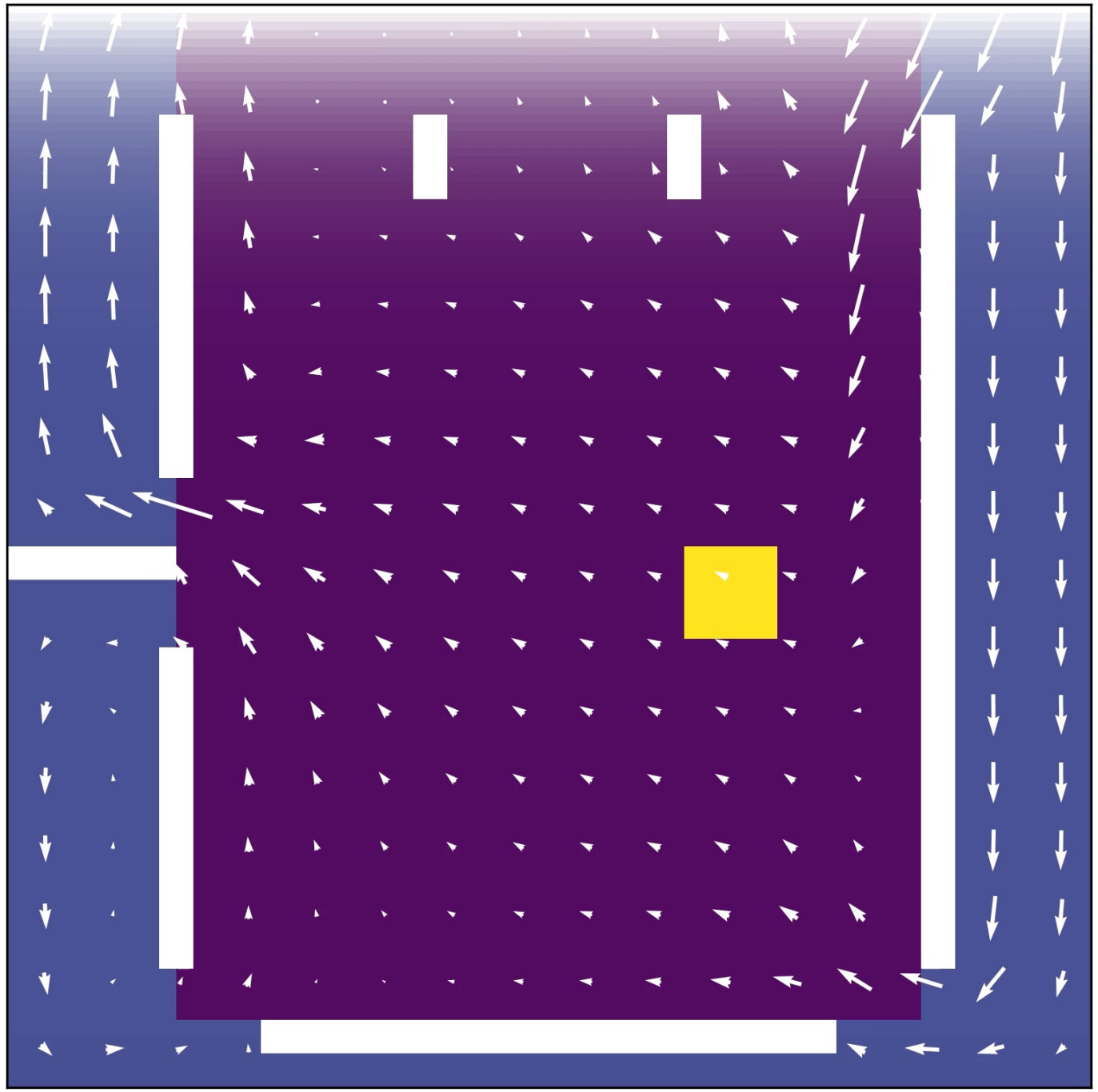

2D Indirect Fluid Control

Source:
Learning to Control PDEs with Differentiable Physics,
Holl et. al

Differentiable Physics Simulation

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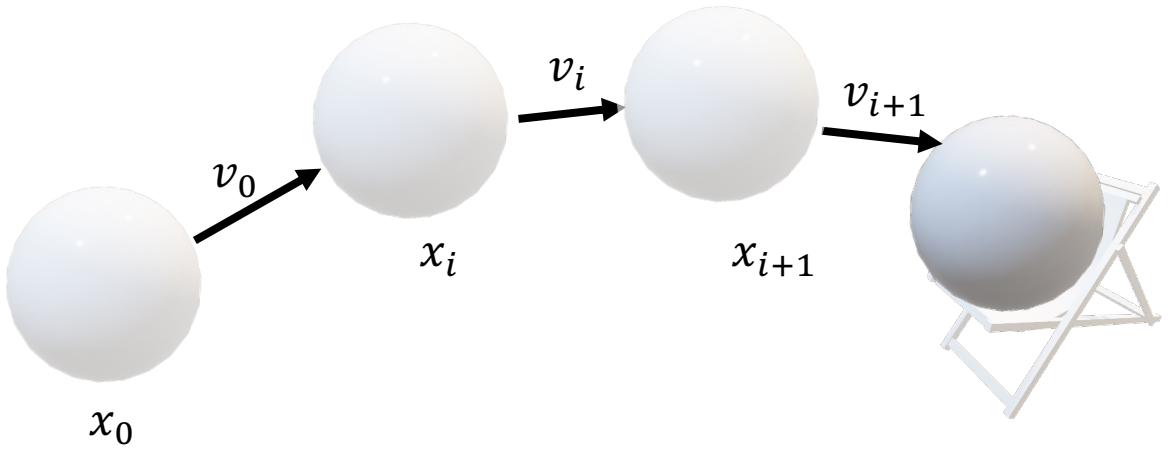
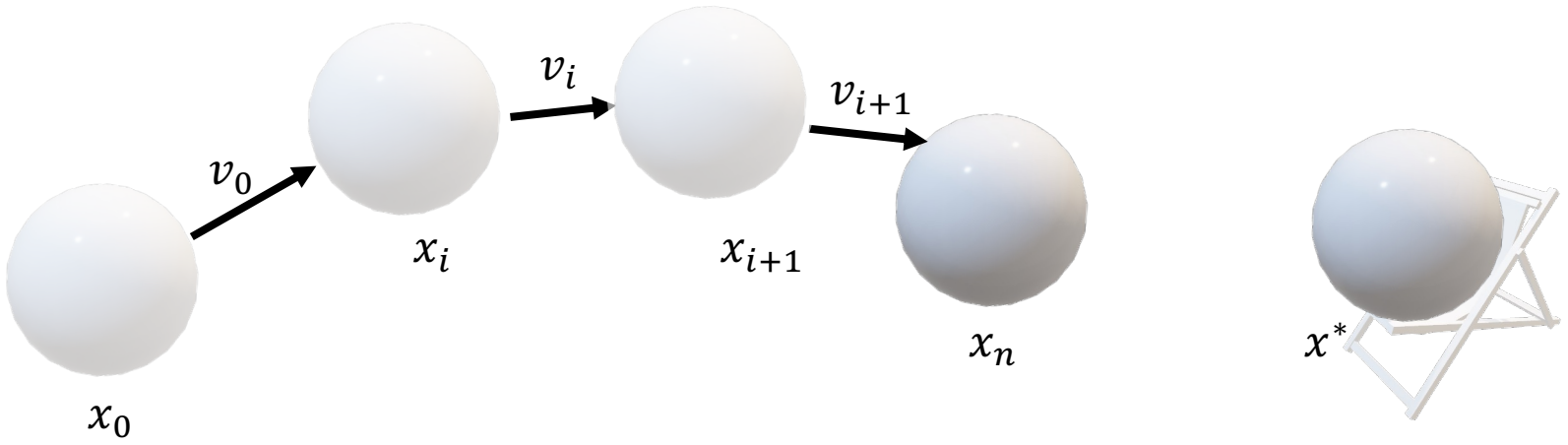


Differentiable Physics Simulation

$L(x_n) = |x^* - x_n|_2^2$ "Loss function"



Differentiable Physics Simulation

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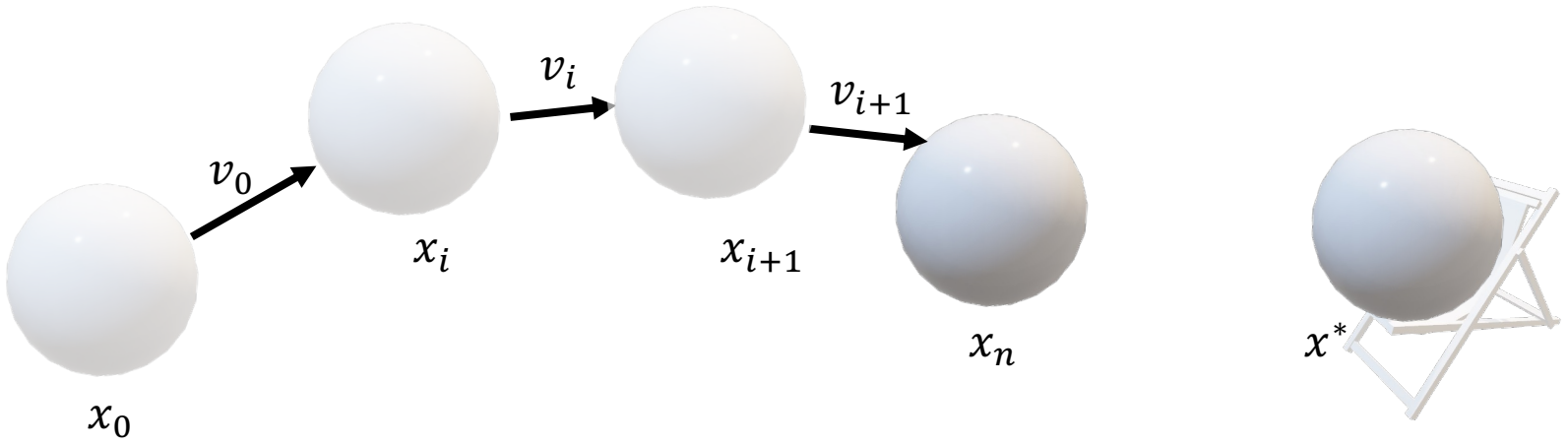
Let's compute $\frac{\partial L}{\partial x_0}$ and $\frac{\partial L}{\partial v_0}$.



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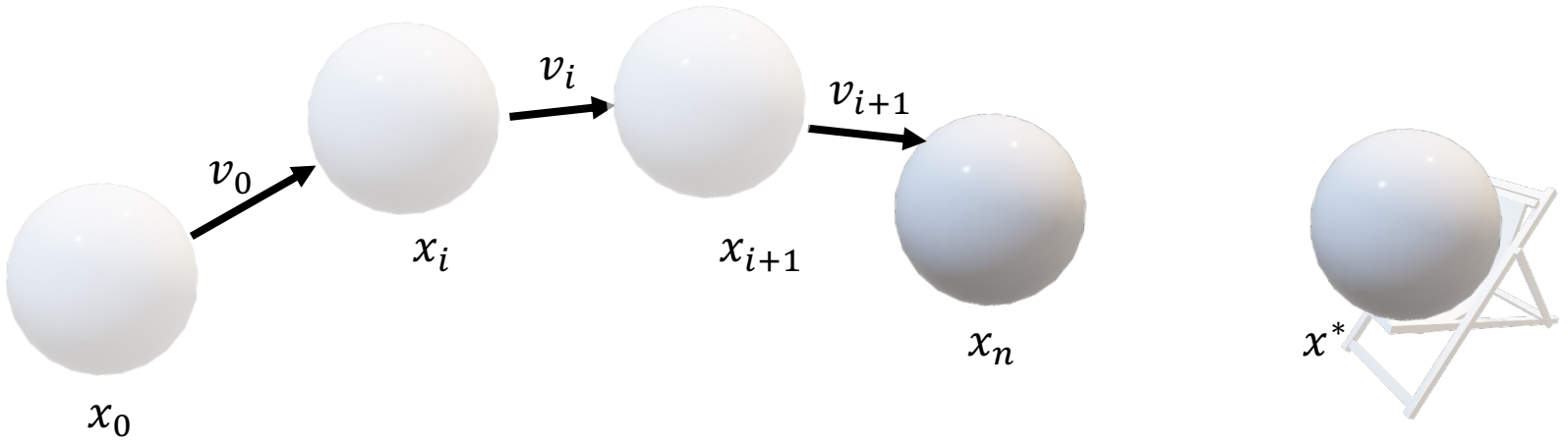

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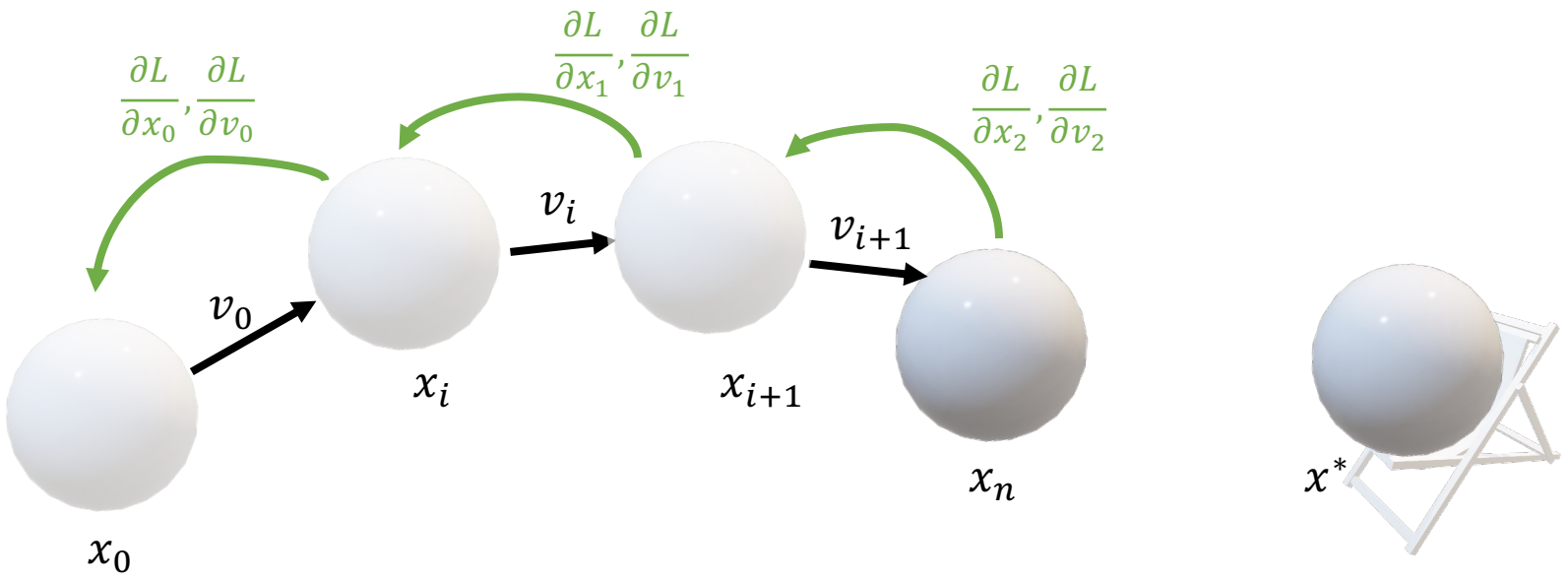
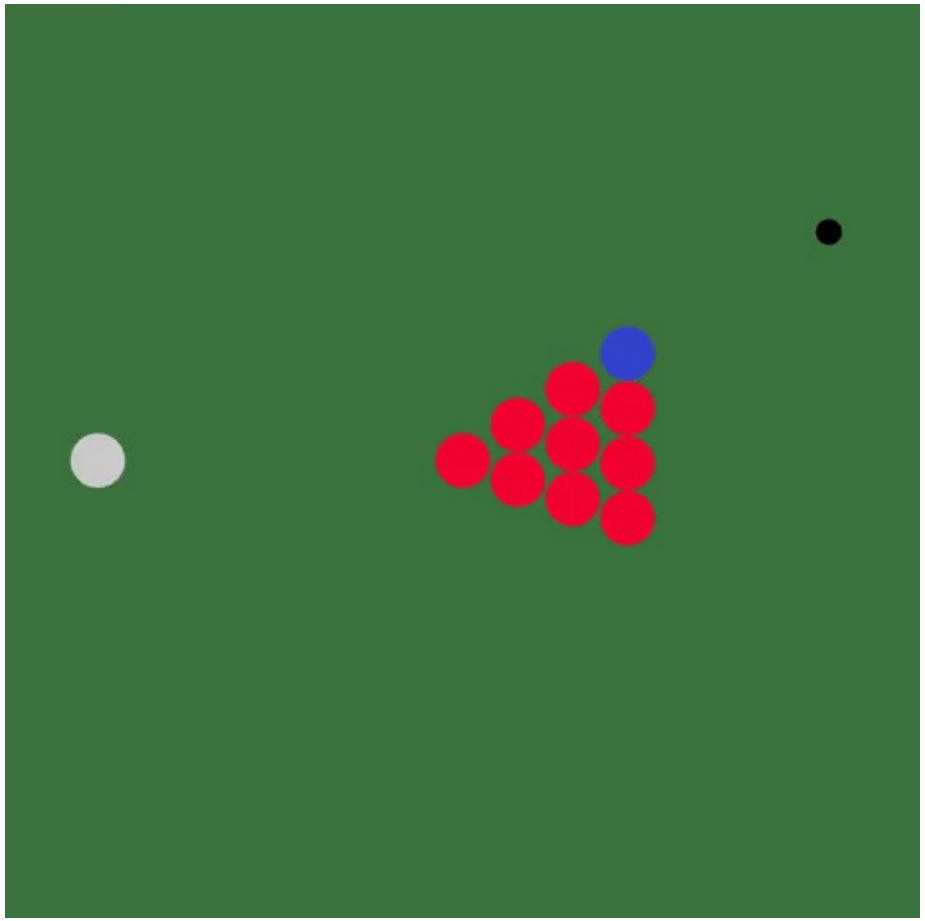
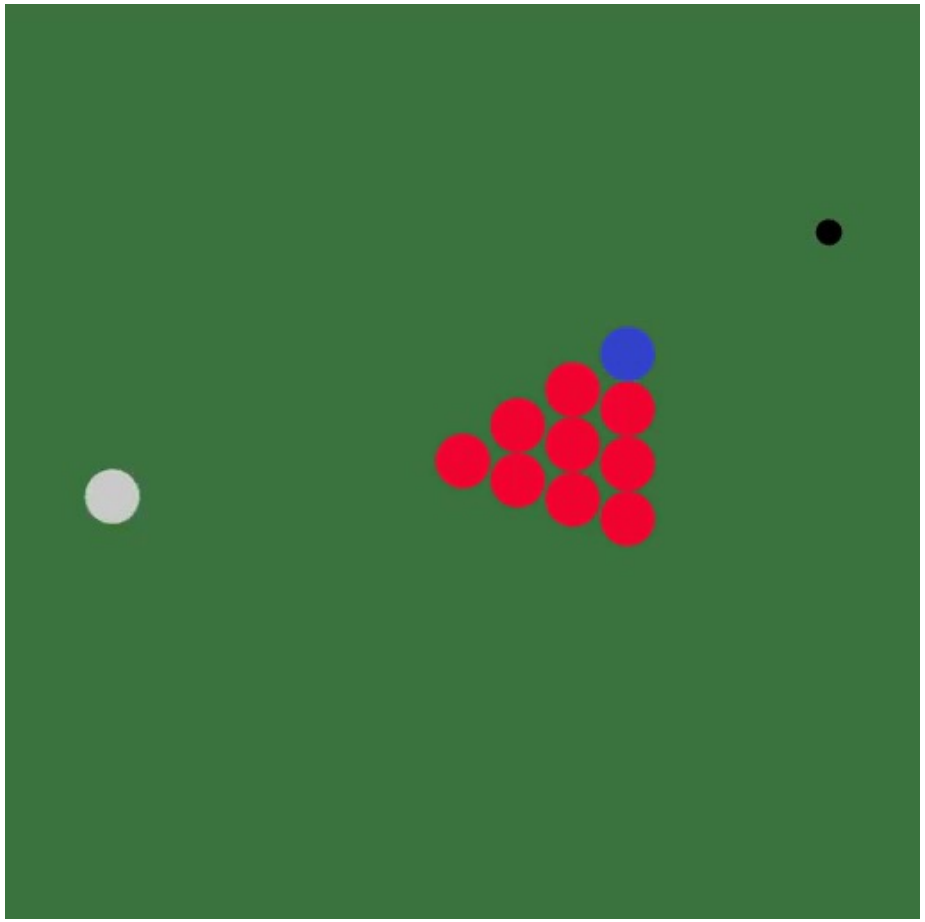




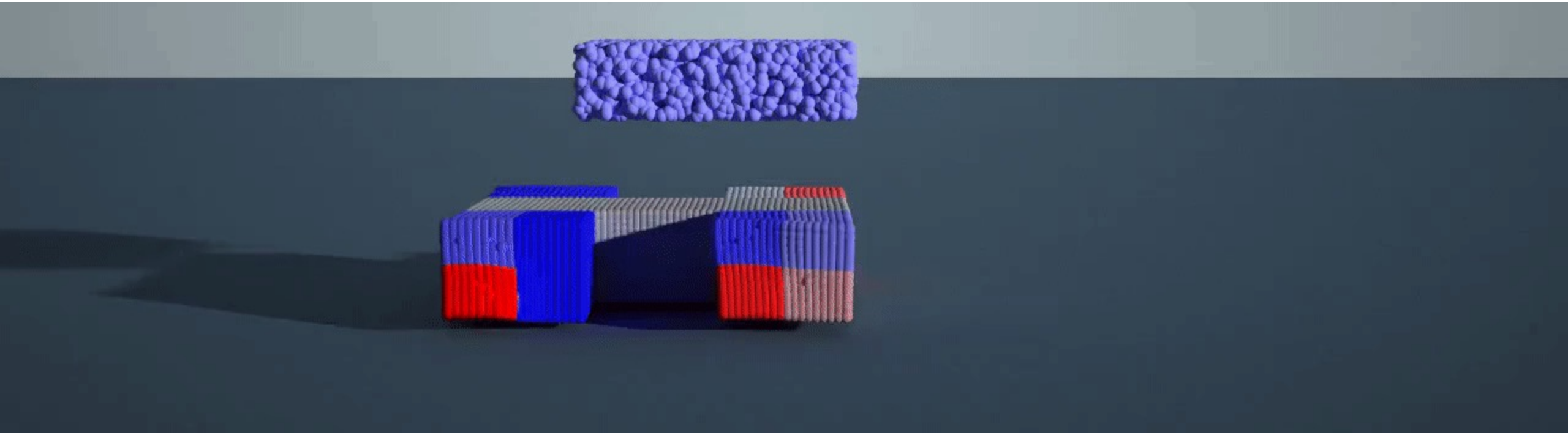
Gradient descent iteration 0 …



… and iteration 100

Source: https://github.com/taichi-dev/difftaichi#differentiable-billiard-simulator-python3-billiardspy

Elastic Object + Fluid
450 gradient descent iteration



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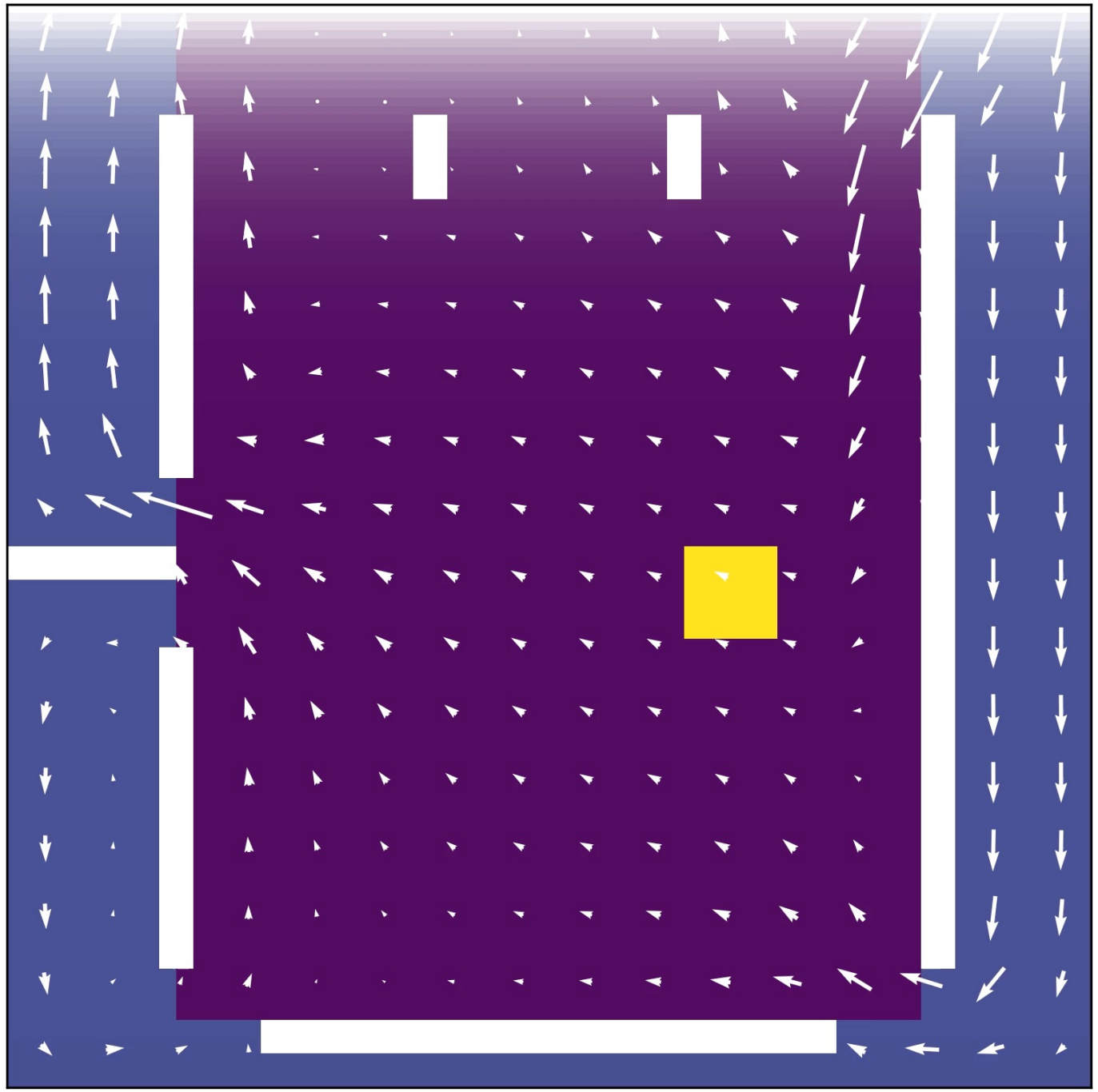


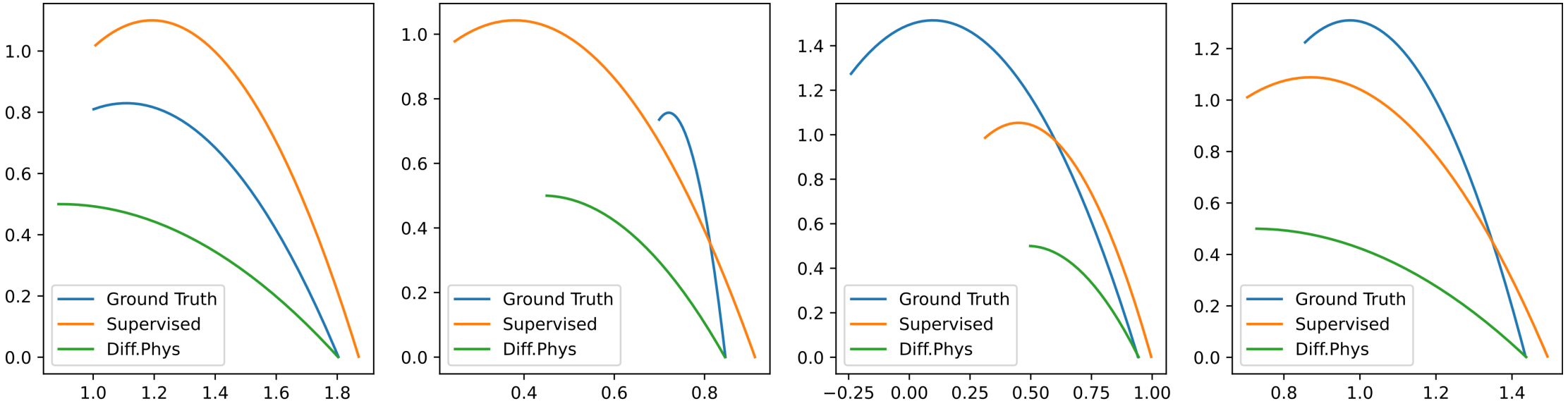
2D Indirect Fluid Control

100 training samples, each with 16 time steps

Learns to move >99% of the volume into the bucket

Source:
Learning to Control PDEs with Differentiable Physics,
Holl et. al

Supervised vs. Diff. Physics loss



Learning to throw, first 4 test cases

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Partial Differential Equations (PDEs)

- The most fundamental description of evolving systems
- From quantum mechanics to turbulent flows
- For a physical system $\boldsymbol{u}(\boldsymbol{x}, t)$

$$\frac{\partial \boldsymbol{u}}{\partial t} = \mathcal{P}\left(\boldsymbol{u}, \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}}, \frac{\partial^2 \boldsymbol{u}}{\partial \boldsymbol{x}^2}, ..., \boldsymbol{y}(t)\right)$$

$$\frac{\partial \boldsymbol{u}}{\partial t} = \mathcal{P}\left(\boldsymbol{u}, \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}}, \frac{\partial^2 \boldsymbol{u}}{\partial \boldsymbol{x}^2}, ...\right) + \boldsymbol{F}(t)$$

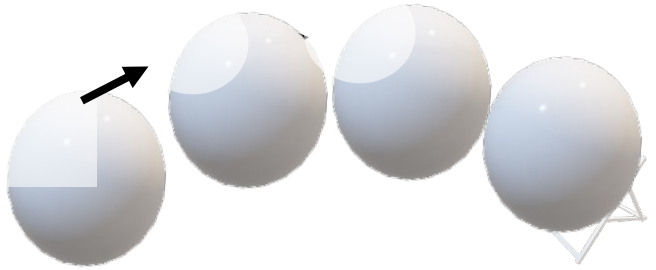
- Agent only has access to $\boldsymbol{o}(\boldsymbol{u})$ (e.g. density, but not velocity)

Partial Differential Equations (PDEs)

$$\frac{\partial \boldsymbol{u}}{\partial t} = \mathcal{P}\left(\boldsymbol{u}, \frac{\partial \boldsymbol{u}}{\partial \boldsymbol{x}}, \frac{\partial^2 \boldsymbol{u}}{\partial \boldsymbol{x}^2}, \ldots\right) + \boldsymbol{F}(t)$$

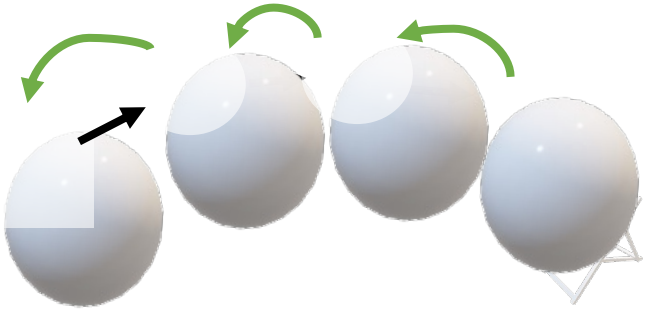
- A Solver moves the system forward by a time increment Δt

$$\boldsymbol{u}(t_{i+1}) = \text{Solver}[\boldsymbol{u}(t_i), \boldsymbol{y}(t_i)] = \boldsymbol{u}(t_i) + \Delta t \cdot \mathcal{P}\left(\boldsymbol{u}(t_i), \ldots, \boldsymbol{y}(t_i)\right)$$



- Differentiable Solvers can efficiently compute the derivatives with respect to any of the inputs:

$$\partial \boldsymbol{u}(t_{i+1}) / \partial \boldsymbol{u}(t_i) \text{ and } \partial \boldsymbol{u}(t_{i+1}) / \partial \boldsymbol{y}(t_i)$$



Goal

i. Find optimal trajectory

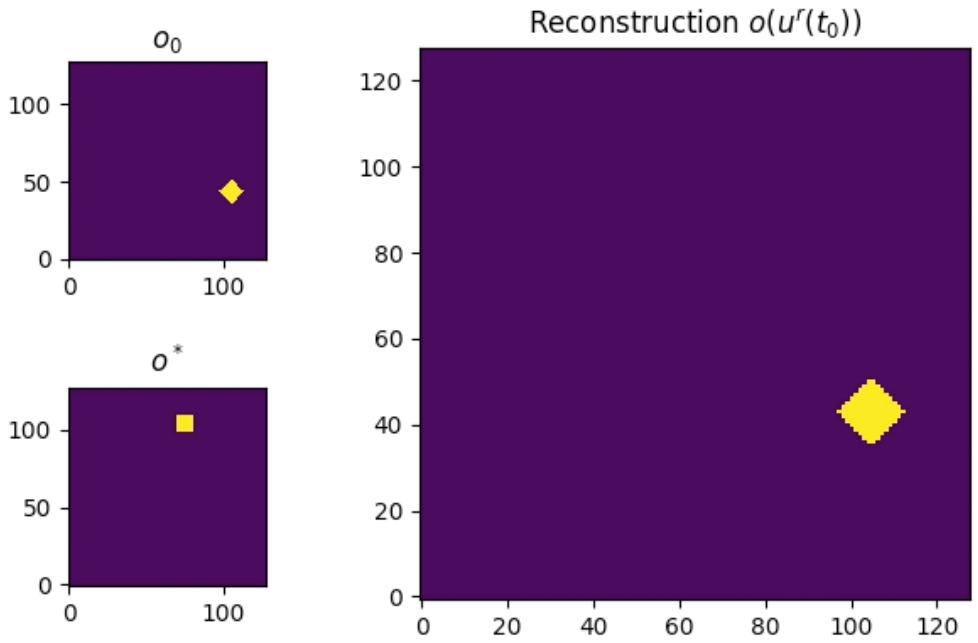
   Observation Predictor (OP) agent

ii. Move the system along it

   Corrector Network agent:

   Control Force Estimator (CFE)

(+ minimize the force applied)



Results

1. Burger’s Equation
2. Incompressible Fluid Flow
    1. Natural fluid flow reconstruction
    2. Shape transition
3. Incompressible Fluid with Indirect Control

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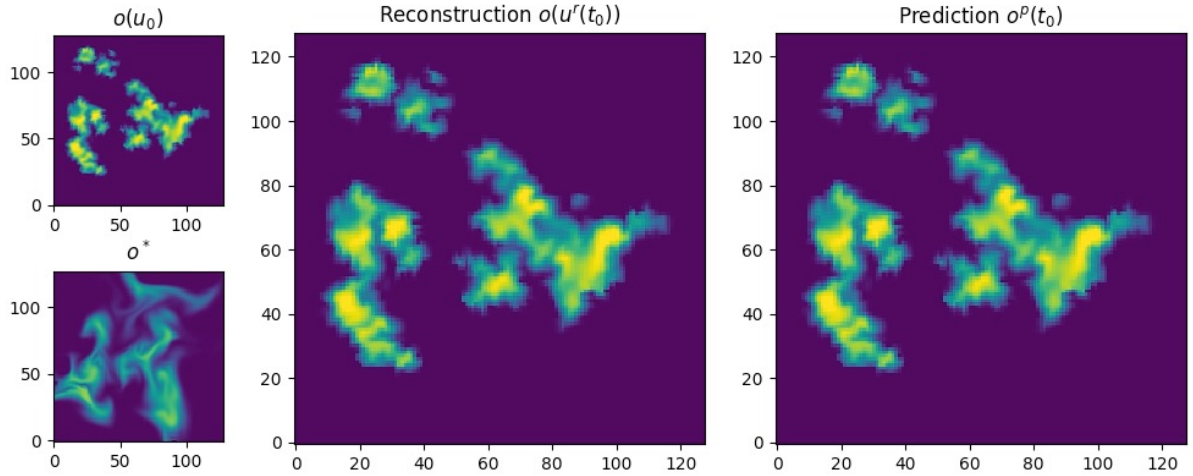

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Neural Networks are Universal Function Approximators

- Numbers in, Numbers out

"AI" → "It's a squirrel"

$$VGG-16\left(\begin{bmatrix}(0.8,0.6,0.15) & \cdots & (0.87\ 0.4,0.2)\\ \vdots & \ddots & \vdots \\ (0.3,0.5,0.4) & \cdots & (0.3,0.7,0.2)\end{bmatrix}\right) = [.02,\ldots,.001,0.4221,\ldots]$$

Neural Networks are Universal Function Approximators

- Pros:
  - Fast
- Con
  - “black box”, hard to explain
  - Unreliable outside known domain
  - Training

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Deep Learning for Cloud Rendering

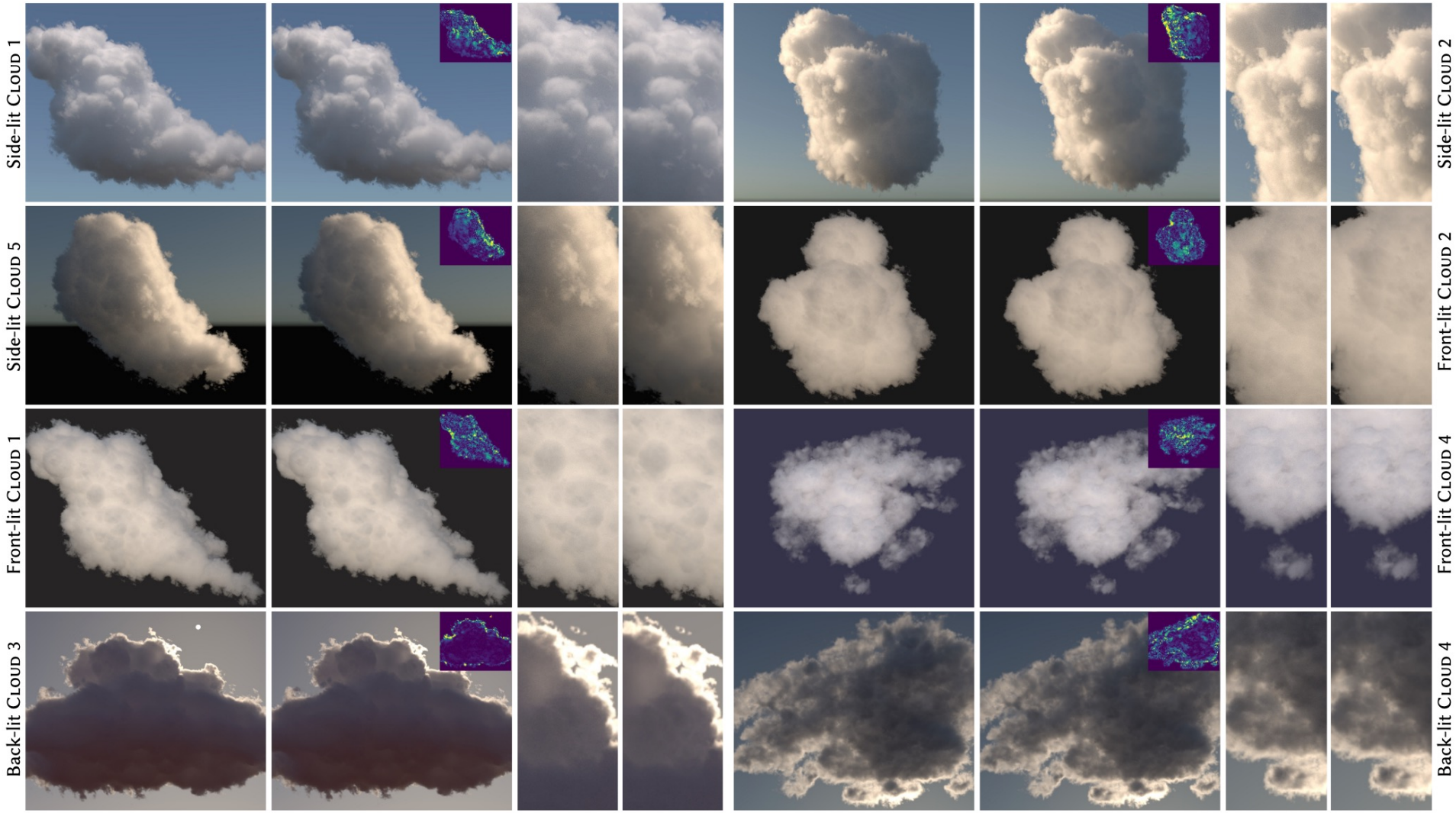
- Approximating the Radiative Transfer Function

$$L(\mathbf{x}, \omega) = \int_0^b T(\mathbf{x}, \mathbf{x}_u)\mu_s(\mathbf{x}_u) \int_{S^2} p(\omega \cdot \widehat{\omega}) L(\mathbf{x}_u, \widehat{\omega})\, \mathrm{d}\widehat{\omega}\, \mathrm{d}u + T(\mathbf{x}, \mathbf{x}_b) L(\mathbf{x}_b, \omega),$$

- By learning the indirect incoming radiance function

$$L_i(\mathbf{x}, \omega) = \int_{S^2} p(\omega \cdot \widehat{\omega})(L(\mathbf{x}, \widehat{\omega}) - L_d(\mathbf{x}, \widehat{\omega}))\, \mathrm{d}\widehat{\omega}.$$

Deep Learning for Cloud Rendering – Results



24x faster convergence than path tracing (PT) on average!

Main takeaway

- Don’t throw away existing knowledge!
- Leverage already existing methods
- Deep Learning **is not magic**, and shouldn’t be treated as such

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Sources

Papers:

- Learning to Control PDEs with Differentiable Physics [Holl et al. 2020] (+ the supplemental materials) https://ge.in.tum.de/publications/2020-iclr-holl/
- DiffTaichi: Differentiable Programming for Physical Simulation https://arxiv.org/abs/1910.00935
- Deep Scattering: Rendering Atmospheric Clouds with Radiance-Predicting Neural Networks, by Kallweit et al. [2017] *https://la.disneyresearch.com/publication/deep-scattering/*

Talks:

- Differentiable Physics (for Deep Learning), Overview Talk by Nils Thuerey *[https://youtu.be/BwuRTpTR2Rg, 40m48s]*
- Differentiable Physical Simulation and AI @ NeurIPS 2020 DiffCVGP workshop *[https://youtu.be/i2O72iMe9ug, 29m59s]*
  - *Differentiable physics and the Taichi programming language*
  - *Code repo with examples: https://github.com/taichi-dev/difftaichi*